1.5 Describe Angle Pair Relationships

Before You used angle postulates to measure and classify angles.

Now You will use special angle relationships to find angle measures.

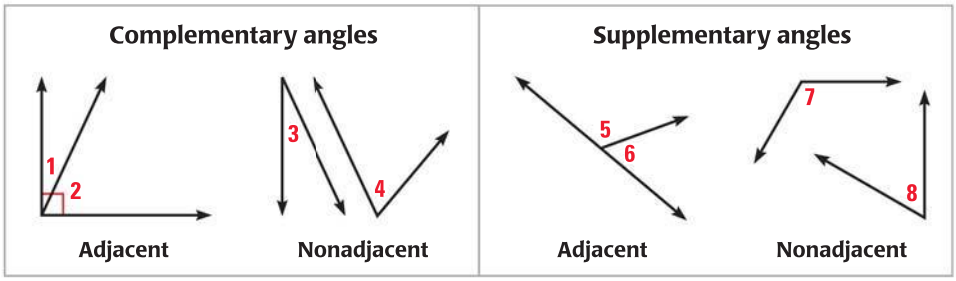
Why? So you can find measures in a building, as in Ex. 53.

Key Vocabulary

- **complementary angles**
- **supplementary angles**
- **adjacent angles**
- **linear pair**
- **vertical angles**

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the *complement* of the other. Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the *supplement* of the other.

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.

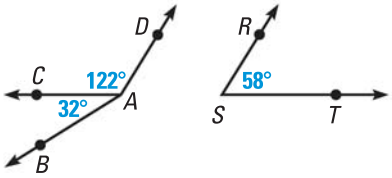


EXAMPLE 1 Identify complements and supplements

AVOID ERRORS

In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex. But they share common interior points, so they are *not* adjacent angles.

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

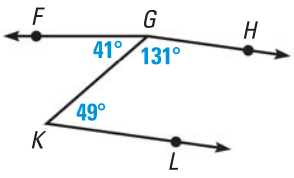
Because $32° + 58° = 90°$, $\angle BAC$ and $\angle RST$ are complementary angles.

Because $122° + 58° = 180°$, $\angle CAD$ and $\angle RST$ are supplementary angles.

Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent.

GUIDED PRACTICE for Example 1

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? *Explain*.

EXAMPLE 2 Find measures of a complement and a supplement

READ DIAGRAMS
Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

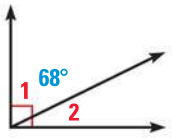
a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68°$, find $m\angle 2$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56°$, find $m\angle 3$.

Solution

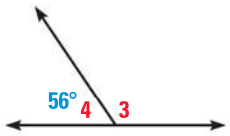
a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$m\angle 2 = 90° - m\angle 1 = 90° - 68° = 22°$



b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

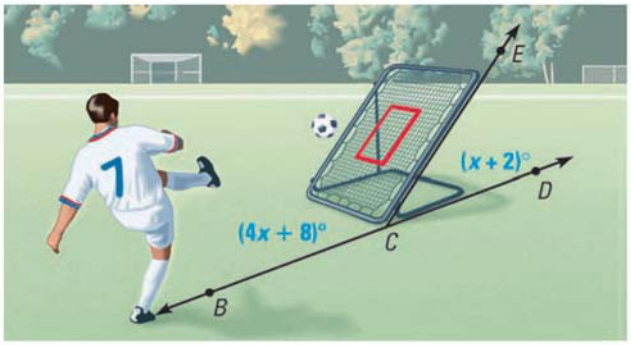
$m\angle 3 = 180° - m\angle 4 = 180° - 56° = 124°$



EXAMPLE 3 Find angle measures

READ DIAGRAMS
In a diagram, you can assume that a line that looks straight *is* straight. In Example 3, B, C, and D lie on $\overleftrightarrow{BD}$. So, $\angle BCD$ is a straight angle.

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



Solution

STEP 1 **Use** the fact that the sum of the measures of supplementary angles is 180°.

$m\angle BCE + m\angle ECD = 180°$ **Write equation.**

$(4x + 8)° + (x + 2)° = 180°$ **Substitute.**

$5x + 10 = 180$ **Combine like terms.**

$5x = 170$ **Subtract 10 from each side.**

$x = 34$ **Divide each side by 5.**

STEP 2 **Evaluate** the original expressions when $x = 34$.

$m\angle BCE = (4x + 8)° = (4 \cdot 34 + 8)° = 144°$

$m\angle ECD = (x + 2)° = (34 + 2)° = 36°$

▶ The angle measures are 144° and 36°.

GUIDED PRACTICE for Examples 2 and 3

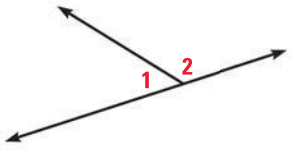
3. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8°$, find $m\angle 1$.

4. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117°$, find $m\angle 4$.

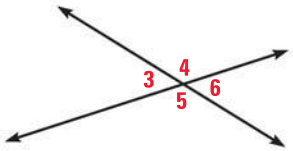
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)°$ and $m\angle PQR = (9x + 1)°$.

ANGLE PAIRS Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.



∠1 and ∠2 are a linear pair.

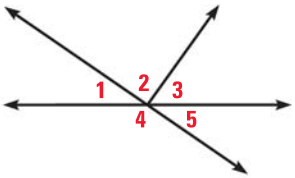


∠3 and ∠6 are vertical angles.
∠4 and ∠5 are vertical angles.

EXAMPLE 4 Identify angle pairs

AVOID ERRORS
In the diagram, one side of ∠1 and one side of ∠3 are opposite rays. But the angles are not a linear pair because they are not adjacent.

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

▶ ∠1 and ∠5 are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ ∠1 and ∠4 are a linear pair. ∠4 and ∠5 are also a linear pair.

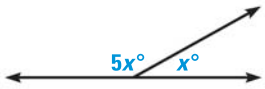
EXAMPLE 5 Find angle measures in a linear pair

xy ALGEBRA Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

Solution

DRAW DIAGRAMS
You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

Let $x°$ be the measure of one angle. The measure of the other angle is $5x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.



$x° + 5x° = 180°$ **Write an equation.**

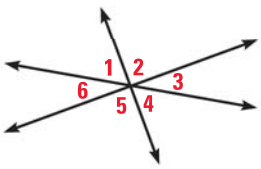
$6x = 180$ **Combine like terms.**

$x = 30$ **Divide each side by 6.**

▶ The measures of the angles are 30° and $5(30°) = 150°$.

GUIDED PRACTICE for Examples 4 and 5

6. Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? *Explain.*

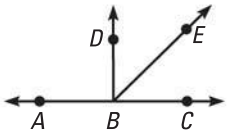


7. The measure of an angle is twice the measure of its complement. Find the measure of each angle.

CONCEPT SUMMARY — For Your Notebook

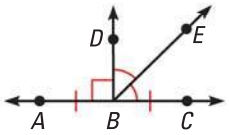
Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you ***can* conclude** from the diagram at the right:



- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point B.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

In the diagram above, you ***cannot* conclude** that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



1.5 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 47
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 30, and 53
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

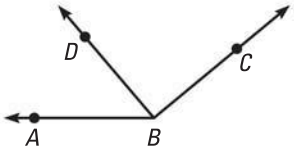
1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain*.

2. ★ **WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain*.

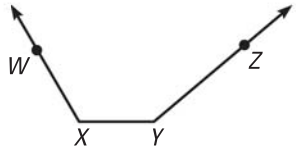
EXAMPLE 1 on p. 35 for Exs. 3–7

IDENTIFYING ANGLES **Tell whether the indicated angles are adjacent.**

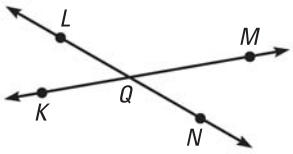
3. $\angle ABD$ and $\angle DBC$



4. $\angle WXY$ and $\angle XYZ$

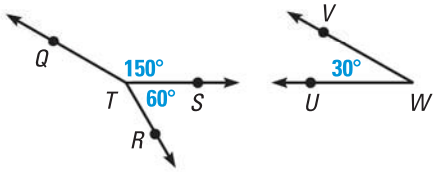


5. $\angle LQM$ and $\angle NQM$

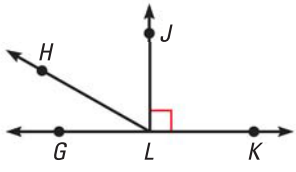


IDENTIFYING ANGLES **Name a pair of complementary angles and a pair of supplementary angles.**

6.



7.



EXAMPLE 2 on p. 36 for Exs. 8–16

COMPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

8. $m\angle 1 = 43°$ **9.** $m\angle 1 = 21°$ **10.** $m\angle 1 = 89°$ **11.** $m\angle 1 = 5°$

SUPPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are supplementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

12. $m\angle 1 = 60°$ **13.** $m\angle 1 = 155°$ **14.** $m\angle 1 = 130°$ **15.** $m\angle 1 = 27°$

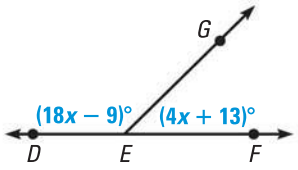
16. ★ **MULTIPLE CHOICE** The arm of a crossing gate moves 37° from vertical. How many more degrees does the arm have to move so that it is horizontal?

Ⓐ 37°
Ⓑ 53°
Ⓒ 90°
Ⓓ 143°

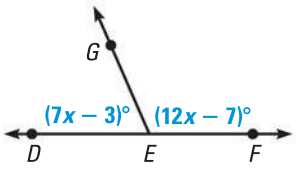
EXAMPLE 3 on p. 36 for Exs. 17–19

ALGEBRA Find $m\angle DEG$ and $m\angle GEF$.

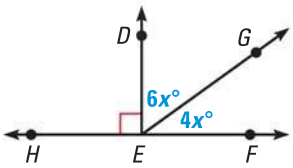
17.



18.



19.



EXAMPLE 4 on p. 37 for Exs. 20–27

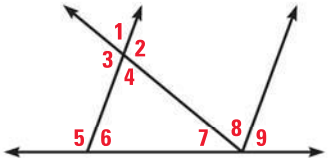
IDENTIFYING ANGLE PAIRS Use the diagram below. Tell whether the angles are *vertical angles*, a *linear pair*, or *neither*.

20. $\angle 1$ and $\angle 4$ **21.** $\angle 1$ and $\angle 2$

22. $\angle 3$ and $\angle 5$ **23.** $\angle 2$ and $\angle 3$

24. $\angle 7$, $\angle 8$, and $\angle 9$ **25.** $\angle 5$ and $\angle 6$

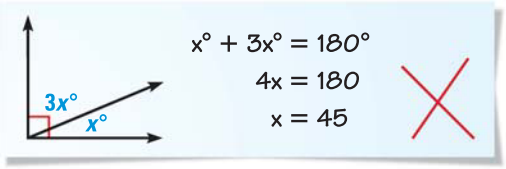
26. $\angle 6$ and $\angle 7$ **27.** $\angle 5$ and $\angle 9$



EXAMPLE 5 on p. 37 for Exs. 28–30

28. **ALGEBRA** Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.

29. **ERROR ANALYSIS** *Describe* and correct the error made in finding the value of x.

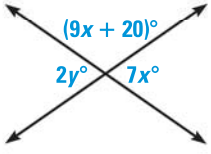


30. ★ **MULTIPLE CHOICE** The measure of one angle is 24° greater than the measure of its complement. What are the measures of the angles?

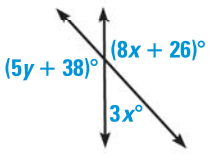
Ⓐ 24° and 66° Ⓑ 24° and 156° Ⓒ 33° and 57° Ⓓ 78° and 102°

ALGEBRA Find the values of x and y.

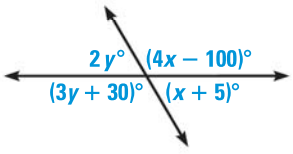
31.



32.



33.



REASONING **Tell whether the statement is *always*, *sometimes*, or *never* true. *Explain* your reasoning.**

34. An obtuse angle has a complement.

35. A straight angle has a complement.

36. An angle has a supplement.

37. The complement of an acute angle is an acute angle.

38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES **$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.**

39. $m\angle A = (3x + 2)^\circ$
$m\angle B = (x - 4)^\circ$

40. $m\angle A = (15x + 3)^\circ$
$m\angle B = (5x - 13)^\circ$

41. $m\angle A = (11x + 24)^\circ$
$m\angle B = (x + 18)^\circ$

FINDING ANGLES **$\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.**

42. $m\angle A = (8x + 100)^\circ$
$m\angle B = (2x + 50)^\circ$

43. $m\angle A = (2x - 20)^\circ$
$m\angle B = (3x + 5)^\circ$

44. $m\angle A = (6x + 72)^\circ$
$m\angle B = (2x + 28)^\circ$

45. **CHALLENGE** You are given that $\angle GHJ$ is a complement of $\angle RST$ and $\angle RST$ is a supplement of $\angle ABC$. Let $m\angle GHJ$ be x°. What is the measure of $\angle ABC$? *Explain* your reasoning.

PROBLEM SOLVING

IDENTIFYING ANGLES **Tell whether the two angles shown are *complementary*, *supplementary*, or *neither*.**

46.

47.

48.

@HomeTutor for problem solving help at classzone.com

ARCHITECTURE **The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example type of the indicated type of angle pair.**

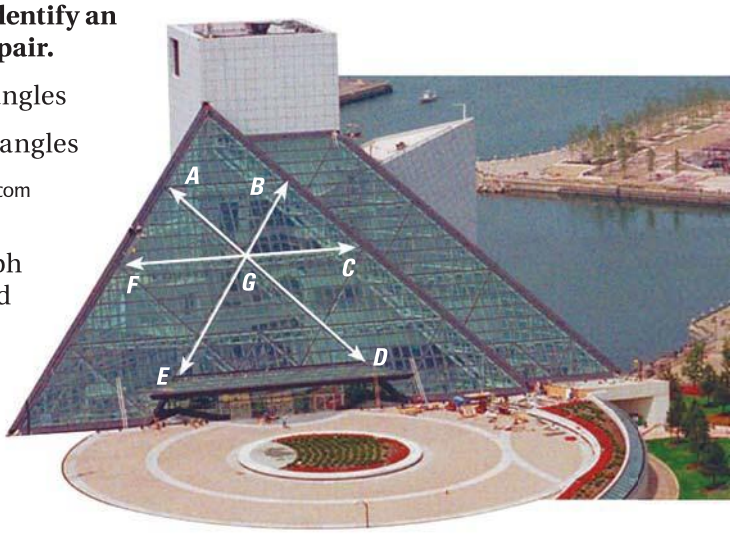
49. Supplementary angles

50. Vertical angles

51. Linear pair

52. Adjacent angles

@HomeTutor for problem solving help at classzone.com



53. ★ **SHORT RESPONSE** Use the photograph shown at the right. Given that $\angle FGB$ and $\angle BGC$ are supplementary angles, and $m\angle FGB = 120^\circ$, *explain* how to find the measure of the complement of $\angle BGC$.

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

= MULTIPLE REPRESENTATIONS

54. SHADOWS The length of a shadow changes as the sun rises. In the diagram below, the length of $\overline{CB}$ is the length of a shadow. The end of the shadow is the vertex of $\angle ABC$, which is formed by the ground and the sun's rays. *Describe* how the shadow and angle change as the sun rises.

55. MULTIPLE REPRESENTATIONS Let $x°$ be an angle measure. Let $y_1°$ be the measure of a complement of the angle and let $y_2°$ be the measure of a supplement of the angle.

a. Writing an Equation Write equations for y_1 as a function of x, and for y_2 as a function of x. What is the domain of each function? *Explain*.

b. Drawing a Graph Graph each function and *describe* its range.

56. CHALLENGE The sum of the measures of two complementary angles exceeds the difference of their measures by 86°. Find the measure of each angle. *Explain* how you found the angle measures.

MIXED REVIEW

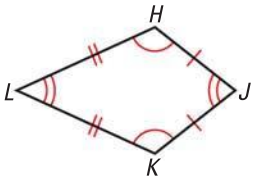
Make a table of values and graph the function. *(p. 884)*

57. $y = 5 - x$ **58.** $y = 3x$ **59.** $y = x^2 - 1$ **60.** $y = -2x^2$

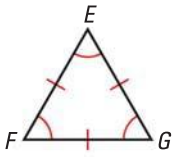
PREVIEW
Prepare for Lesson 1.6 in Exs. 61–63.

In each figure, name the congruent sides and congruent angles. *(pp. 9, 24)*

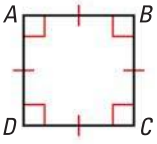
61.



62.



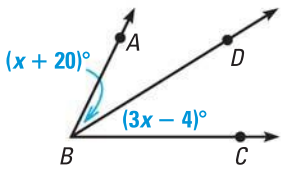
63.



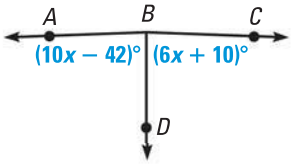
QUIZ *for Lessons 1.4–1.5*

In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$. *(p. 24)*

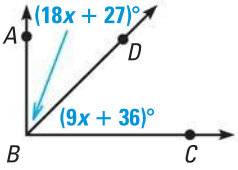
1.



2.



3.



Find the measure of (a) the complement and (b) the supplement of $\angle 1$. *(p. 35)*

4. $m\angle 1 = 47°$ **5.** $m\angle 1 = 19°$ **6.** $m\angle 1 = 75°$ **7.** $m\angle 1 = 2°$